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Profitability and Market Quality of High Frequency Market-makers: An Empirical Investigation[1]

By

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Profitability and Market Quality of High Frequency Market Makers : An Empirical Investigation

Abstract

Financial markets in contemporary regulatory settings require the presence of high-frequency liquidity providers. We present an applied study of the profitability and the impact on market quality of an individual high-frequency trader acting as a market-maker. Using a sample of sixty stocks over a six-month period, we implement the optimal quoting policy (OQP) of liquidity provision from Ait-Sahalia and Saglam (2014) dynamic inventory management model. The OQP allows the high-frequency trader to extract a constant annuity from the market but its profitability is insufficient to cover the costs of market-making activities. The OQP is embedded in a trading strategy that relaxes the model's constraint on the quantity traded. Circuit-breakers are implemented and market imperfections are considered. Profits excluding maker-fees and considering transaction fees are economically significant. We propose a methodology to adjust the returns for asynchronous trading and varying leverage levels associated with dynamic inventory management. This allows us to qualify high trade volume as a proxy of informed trading. The high-frequency trader behaves as a constant liquidity provider and has a positive effect on market quality even in periods of market stress.

Keywords: algorithmic trading, electronic markets, high-frequency trading, limit order book, liquidity, market-making, market efficiency, market microstructure

JEL classification : G10, G12, G14

1. Introduction

Most stock exchanges have removed or have diluted the formal obligation to maintain an orderly market once imposed on human market-makers: high-frequency liquidity suppliers are major participants in electronic markets (Anand and Venkataraman (2013); Menkveld (2013)). Jones (2013) explains the increase in high-frequency market making by lower cost structures and more adequate responses to adverse selection.

Notwithstanding the importance of high-frequency market-making, very little is known about the profitability and individual behavior of high-frequency liquidity providers. Menkveld (2013) describes and evaluates the activities of a large high-frequency market maker (HFMM) who uses spatial arbitrage as the core of his market-making strategy. He asserts that fees are a substantial part of the HFMM's profit and loss account. Serbera and Paumard (2016) argue that maker-fees represent the core profitability of high-frequency market-making. Popper (2012) states that profits in American stocks from high-speed trading in 2012 are down 74 percent from the peak of $4.9 billion in 2009. This can be linked to a decrease in commission and rebates, reported by Malinova and Park (2015).

To deal with this trend, we assess the economic viability, excluding maker fees, of a typical HFMM in two steps: first, we emulate the behavior of an HFMM using Ait-Sahalia and Saglam (2014) dynamic inventory management model (the model hereafter). Their model mimics the high frequency trading stylized facts. Their setup differs from the classical dynamic inventory models (Grossman and Stiglitz (1980); Roll (1984); Glosten and Milgrom (1985); Kyle (1985)) in that the strategic variable is whether or not to quote rather than change the supply curve. It yields to an optimal quoting policy (OQP) of liquidity provision that drives the HFMM's trading decisions. Second, we embed the OQP in a trading strategy that relaxes some of the model's assumptions and adds risk management features.

Market-making implies asynchronous trades and varying market risk related to the dynamic leverage from the model's OQP and the liquidity demanders' needs. These factors affect the HFMM's trading performance. We propose a measure, the time-volume weighted average return (TVWAR), to cope with both phenomena. It allows us to analyze a single stock performance incurring different phases of trading activities and/or liquidity depth, and to compare returns of stocks with different idiosyncratic characteristics.

The emulation provides insights into the implications of high-frequency market-making on market quality, a matter that has raised much concern. Duffie (2010) describes the importance of monitoring the pattern of response to supply and demand shocks for asset pricing dynamics. Foucault, Kadan, and Kandel (2013) develop a model based on an endogenous reaction time to trading activities, and find that algorithmic trading plays an important role in monitoring the state of liquidity cycles. Biais, Foucault, and Moinas (2015) and Pagnotta and Philippon (2015) analyze competition on speed. They argue that competition should have a positive effect on the price discovery process. Finally, market stability is documented using Johnson et al. (2013) ultrafast extreme events (UEEs).

The paper is organized as follows. Section 2 presents the dynamic inventory management model of Ait-Sahalia and Saglam (2014) and its optimal quoting policy. Section 3 introduces the empirical investigation. Section 4 presents the data. Section 5 proposes a measure to determine the returns in a context of asynchronous data and dynamic inventory management. Section 6 sets out and discusses the results. Section 7 presents robustness tests, and Section 8 concludes the paper.

2. Optimal Quoting Policy of Liquidity Provision

Ait-Sahalia and Saglam (2014) refer to two types of agents: low-frequency traders (LFTs), who use market orders only, and a sole HFMM who has exclusive access to the limit order book (LOB). The HFMM trades limit orders (LOs) only, and exhibits inventory aversion. The bid-ask spread is exogenous. This setup differs from the classical dynamic inventory models in that the strategic variable is whether or not to quote and not to change the HFMM's supply curve. The HFMM's revenue depends on the trade-off between the inflows from the bid-ask spread and the outflows from the inventory cost as depicted in the following equation:

$$E(\pi) = \frac{C}{2}\sum_{t=1}^{\infty} e^{-Dsmo_t} I\left(l_{smo_t}^{b} = 1\right) + \frac{C}{2}\sum_{t=1}^{\infty} e^{-Dbmo_t} I\left(l_{bmo_t}^{a} = 1\right) - \Gamma\int_{0}^{\infty} e^{-Dt}|x_t|\, dt, \qquad \textbf{(1)}$$

where:

$E(\pi)$: Quoting policy expected reward.
C: Bid-ask spread.
D: Constant discount factor > 0.
smo_t (bmo_t): Sell (buy) market order by LFTs at time *t*.
I: Indicator function.
b: HFMM bid limit order.
a: HFMM ask limit order.
$l_{smo_t|bmo_t}^{b|a}$: Equals 1 if the HFMM is quoting a bid (b) or an ask (a) limit order when a LFT sell (buy) market order arrives, 0 otherwise.
Γ: Inventory aversion coefficient.
x_t: Inventory position at time *t*.

The first term to the right of equation **(1)** is the discounted value of the HFMM's revenue $\left(\frac{C}{2}\right)$ earned when an incoming LFT's sell market order hits the HFMM's limit order while he is bidding $\left(I\left(l_{smo_t}^{b} = 1\right)\right)$. The second term is the discounted revenue associated with an incoming LFT's bid market order, and the third term is the discounted value of the HFMM's inventory costs over the period dt. To keep the model tractable, the HFMM always places his LOs at the best bid and/or ask price and does not issue orders larger than one contract.

Apart from observing the arrival of market orders, the HFMM receives a signal s about the likely side of the next incoming market order: $s \in \{1, -1\}$, where 1 predicts an incoming LFT's buy market order and -1 an incoming LFT's sell market order. P quantifies the informational quality of the HFMM's signal. It varies from 0.5 (no prior knowledge about the side of the next incoming LFT's market order) to 1.0 (perfect knowledge). In Ait-Sahalia and Saglam (2014) setup, the next event is either 1: the arrival of a signal with probability $\left(\frac{\mu/2}{\lambda+\mu}\right)$, μ being the arrival rate of a Poisson distribution of the HFMM's signals and λ the arrival rate of a Poisson distribution of the incoming LFTs' market orders; 2: the arrival of a market order in the direction of the last signal with probability $\frac{P\lambda}{\lambda+\mu}$; or 3: the arrival of a market order in the opposite direction of the last signal with probability $\frac{(1-P)\lambda}{\lambda+\mu}$. The value of market-making activities for any given event assuming an inventory position of x $(x \in \{\cdots, -2, -1, 0, 1, 2, \cdots\})$ and a sell signal (-1) is:

$$v(x,-1) = \tag{2}$$

$$-\gamma|x| + \delta \left\{ \begin{matrix} \left(\frac{\mu/2}{\lambda+\mu}\right)\left(v(x,1)+v(x,-1)\right) + \frac{P\lambda}{\lambda+\mu} max\left(\frac{c}{2\delta} + v(x-1,-1), v(x,-1)\right) + \\ \frac{(1-P)\lambda}{\lambda+\mu} max\left(\frac{c}{2\delta} + v(x+1,-1), v(x,-1)\right) \end{matrix} \right\},$$

where:

$\gamma = \frac{\Gamma}{\lambda+\mu+D}$; $\delta = \frac{\lambda+\mu}{\lambda+\mu+D}$; $c = \delta C$;

Equation **(2)** quantifies the market-making value function. The first term to the right is the discounted inventory cost $(-\gamma|x|)$. The second term is the discounted value of the three possible events: the value of the arrival of a signal $\left(\left(\frac{\mu/2}{\lambda+\mu}\right)\left(v(x,1)+v(x,-1)\right)\right)$, the value of the arrival of a market order in the direction of the signal $\left(\frac{P\lambda}{\lambda+\mu} max\left(\frac{c}{2\delta} + v(x-1,-1), v(x,-1)\right)\right)$, and the value of the arrival of a market order in the opposite direction of the signal $\left(\frac{(1-P)\lambda}{\lambda+\mu} max\left(\frac{c}{2\delta} + v(x+1,-1), v(x,-1)\right)\right)$.

Solving equation **(2)** by backward induction using the Hamilton-Jacobi-Belman optimality method leads to the optimization of the expected reward trade-off.

2.1 OQP determination

Theorem 1 of Ait-Sahalia and Saglam (2014) states that there is an optimal quoting policy of liquidity provision, based on the expected reward trade-off:

Theorem 1: The optimal quoting policy of the HFMM consists in quoting at the best bid and the best ask according to a threshold policy, i.e., there exists $L^* < 0 < U^* \leq |L^*|$, such that:

$$l^b(x,1) = \begin{cases} 1 \; when \; x < U^* \\ 0 \; when \; x \geq U^* \end{cases} \qquad l^a(x,1) = \begin{cases} 1 \; when \; x > L^* \\ 0 \; when \; x \leq L^* \end{cases}$$
$$l^b(x,-1) = \begin{cases} 1 \; when \; x < -L^* \\ 0 \; when \; x \geq -L^* \end{cases} \quad l^a(x,-1) = \begin{cases} 1 \; when \; x > -U^* \\ 0 \; when \; x \leq -U^* \end{cases}$$

Theorem 1 can be interpreted as follows: Suppose the HFMM receives a “buy” signal ($s = 1$) while being long ($x > 1$). He is going to act upon it ($l^b = 1$) as long as his current inventory is not already too high ($x < U^*$). If ($x \geq U^*$), the HFMM will not quote because this could increase his long inventory position beyond the optimal threshold U^*. Symmetrically, if the HFMM receives a “sell” signal ($s = -1$), he will quote on the ask side ($l^a = 1$) as long as his inventory position is not already too short ($x > -U^*$).

An algorithm proposed by Ait-Sahalia and Saglam (2014) presented in the Appendix allows us to determine the thresholds based on the expected reward trade-off and *Theorem 1*.

2.2 Emulation and OQP

To determine an optimal quoting policy of liquidity provision, the model requires six parameters: $D, \Gamma, \lambda, \mu, C, P$. All financial instruments use the same and constant parameters D, the discount rate, and Γ, the coefficient of inventory aversion.

The four remaining parameters depend on the idiosyncratic behaviors of the stocks. C is the observed bid-ask spread and λ, the observed arrival rate of marketable orders. We define μ, the HFMM’s arrival rate of signals, as the number of creations, updates, and cancellations at the LOB level 1. We constrain the HFMM to react to other market participants’ actions. He does not use any private information to modify the observed price discovery process and/or the bid-ask spread. The parameter P is fixed at 0.50.

For any given combination of the six input parameters, we obtain an ex-ante OQP of liquidity provision based on the algorithm described in the Appendix. The algorithm stipulates the sides (bid and/or ask) and respective quantities to quote, i.e. the thresholds.

3. Empirical investigation

We aim to provide an empirical investigation of the profitability and the impact on market quality of an individual high-frequency trader acting as a liquidity provider. The decision to quote or to trade, the timing and the management of positions are totally driven by our fully automated algorithm. Our approach is fundamentally different from the traditional trading strategy approaches such as Fibonacci ratios, golden ratio, oscillators and pivot point strategies that try to forecast the future value of a financial instrument. Our method involves using the optimal quoting policy from Ait-Sahalia and Saglam (2014) as a kernel. The OQP is independent from the market states and does not require any prediction of prices.

Data mining and data snooping have been analyzed extensively (Wasserstein and Lazar (2016), Bailey et al. (2015), Kim and Ji (2015), and Bailey et al. (2014)). Multiple-testing increases the probability of a false discovery drastically because it takes on average as few as $1/_{\alpha}$ independent iterations to produce a false discovery (Lopez De Prado (2015)). Our results are obtained following a single set of parameter values designed ex-ante and therefore do not imply any data mining or data snooping.

First, we assess the performance of the optimal quoting policy from Ait-Sahalia and Saglam (2014). However, we impose the closing of all positions by issuing market orders at the end of the day (EOD); the procedure is launched at the beginning of the last three minutes of trading. The appraisal thus represents the results of "pure" market-making as accurately as possible.

Second, we embed the OQP in a trading strategy that considers market imperfections: limit orders are not uniquely identified in our database. Usually we cannot know with certainty who holds time priority. We apply the worst-case scenario to the HFMM: time priority is given to the total quantity available at the best bid (ask), excluding the HFMM limit order, one microsecond (µS) before the arrival of a market order. In this way, we depart from Ait-Sahalia and Saglam (2014), who assume that the HFMM is the fastest trader.

In practice, trading firms monitor market conditions and integrate pauses in their algorithms (Kelejian and Mukerji (2016)). Events like the flash crash of May 2010 and the Knight Capital's algorithm glitch of August 2012[2] prompted regulators such as the Commodity Futures Trading Commission (2013), the U.S. Securities and Exchange Commission (2016), and tThe Government Office for Science (2012)) to make the use of circuit-breakers mandatory. We enforce circuit-breakers by monitoring market conditions associated with three of the OQP's parameters: λ and μ have an upper bound corresponding to 95% of the ranges of values from the reference time interval of one minute. C, the bid-ask spread, has an upper bound of 99% of the reference range. When a parameter's value exceeds its upper bound, we cancel all the quotes on the stock and we send a marketable order to liquidate the position. Parameter values are reset to zero at the beginning of the next time interval. This induces regular quoting and trading activities. This behavior is in line with Chordia et al. (2013), who note that market-makers are also liquidity takers in their regular activities.

Within the model of Ait-Sahalia and Saglam (2014), the quantity of each order is fixed at one lot. To relax the constraint imposed on profits, we generalize this concept by defining κ, a constant quantity. κ is similar in nature to the trading unit of an option contract, e.g. 100 stocks, and is defined as the maximum quantity from the five most frequently traded quantities of a given stock. The fragmentation of orders is a well-established concept (Almgren and Chriss (2001); Almgren (2003); Obizhaeva and Wang (2013); Markov (2014); Jingle and Phadnis (2013); among others). Choosing κ with the suggested methodology reflects the fact that market participants want to mitigate their impact on the price discovery process. This is supported by the statistics of Table 2, which demonstrate that 47.6% (41.2%) of all trades in the DAX (MDAX) do not consume the available quantity at level one.

Trading a quantity larger than 1 could cause a price impact if the available quantity at level one is insufficient to liquidate the HFMM's position. This would force the HFMM to walk into the limit order book. Empirical investigations of trading strategies are vulnerable to biases if they exhibit price impacts. This could be an indication of an undue influence on the price discovery process and it could lead real-time trading results to differ significantly from expectations. To avoid HFMM's market orders and the implied illiquidity cost transfer to other market participants that

[2] http://www.bloomberg.com/bw/articles/2012-08-02/knight-shows-how-to-lose-440-million-in-30-minutes

affect prices, the HFMM incurs market risk instead of walking up (down) the LOB. The HFMM trades up to the available quantity at level one and waits for the next incoming limit order(s) at that level in order to fully liquidate his position if necessary. This amounts to controlling for the instantaneous price impact (Cont, Kukanov, and Stoikov (2014); Bouchaud, Farmer, and Lillo (2009)). Estimating the permanent impact of market orders (Hautsch and Huang (2012); Huh (2014); Zhou (2012)) becomes unnecessary. We apply to κ the OQP thresholds associated with the contemporary model's parameters. In case of partial execution of a limit order, we cancel the order(s) and submit a new order(s) with the required adjusted quantity(ies). Time priority is amended accordingly.

Speed is important to gain time priority and to avoid being picked up (sniped) while displaying stale quotes, so we take into consideration the effect of latency on the trading results. We use the latency of 150 μS. This value is representative of the time required by our infrastructure and our algorithm to receive, analyze, react to, and send new orders following the arrival of new information. We can compete on speed with the other co-location firms, and we are significantly faster than buy-side investors.

4. Data

The data come from Xetra, the fully electronic trading platform of the Frankfurt Stock Exchange. The raw dataset contains all events (deltas and snapshots) sent through the Enhanced Broadcast System, a data feed used by high-frequency traders. Deltas track all possible events in the LOB whereas snapshots convey information about the state of a given LOB at a specific time. Xetra Parser, developed by Bilodeau (2013), is used to reconstruct the real-time order book sequence using Xetra protocol and Enhanced Broadcast. Liquidity is provided by market participants posting limit orders in the LOB. The stocks of our sample have LOB with twenty levels on both sides of the market. The state of the LOB and the arrival of marketable orders (the trades) can be observed by the subscribers to the data feed. Time stamps are in μS, trading is anonymous and specific order identification is nonexistent.

Our data set consists of sixty stocks from the DAX index family: thirty stocks in each of the DAX and the MDAX. DAX indexes are indicators for the German equity market. The DAX characterizes the blue chip segment. Its components are the largest and most actively traded German companies. The MDAX is composed of mid-capitalization issues from traditional sectors, excluding technology, that rank immediately below the DAX stocks. The ultra-high-frequency data cover six months from February 1, 2013 to July 31, 2013. The sample covers different market phases as depicted by Figure 1: a trading range for the entire month of February, a bull trend from the last week of April to mid-May, 2 bear trends (mid-March to mid-April and the last week of May to the last week of June) and high volatility periods: the third week of April and the third week of June. Figure 1 displays the DAX daily chart for this period.

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Figure 1 DAX daily quotes - February to July 2013
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Table 1 presents the summary statistics from trading and LOB level one quoting activities. In Panel A, the DAX largely dominates the MDAX with a market value traded of €398.5b (92.57% of total activities) and 17.6 m of transactions (79.47%). Panel B exhibits even stronger statistics for the DAX. Quoting based exclusively on level one activity overwhelms trading as depicted by the ratio of the number of updates to the number of trades (# UTDs/# trades) that is higher than 10 for both indexes. This ratio is followed by the SEC (MIDAS, Security and Exchange Commission at http://www.sec.gov/marketstructure/midas.html) to monitor high-frequency trading activities.

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Table 1 Market summary statistics, trades and LOB[1]
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Table 2 illustrates the price discovery process. Price impact minimization is the dominant trading phenomenon, with 47.6% (DAX) and 41.2% (MDAX) of all trades executed at the last tick price. This includes combinations (0,0), (+,0), and (-,0). Aggressive orders (+,+ and -,-) induce positive autocorrelations in the price discovery process. They represent 14.3% (DAX) and 18.4% (MDAX) of all price moves, less than bid-ask bounce trades (+,- and -,+), which are respectively 18.1% (DAX) and 21.0% (MDAX).

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Table 2 Two-way classification of price movements in consecutive intraday trades: Summary (,000)

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Table 3 exhibits the two-way classification for the thirty stocks from the DAX index. The directional trading maxima (+,+ and -,-) are respectively 10.72% and 11.04% for the unique identifier (isix) 1634, and the other aggregated results are representative of all stocks.

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Table 3 Two-way classification of price movements in consecutive intraday trades DAX (%)

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Results for the thirty stocks from the MDAX are not qualitatively different. They are not presented due to space considerations, but are available upon request.

Johnson et al. (2013) proposed the concept of ultrafast extreme events (UEEs). UEEs can shed light on the price discovery process and the instabilities of financial markets, and help one appraise the HFMM's risk exposure, and stress-test trading algorithms. We define UEEs as an occurrence of a stock price ticking down (up) at least five times before ticking up (down), having a price change of at least 0.5% within a duration of 1500 milliseconds. We can interpret UEEs as surges for up ticks and mini crashes for down ticks. As depicted in Table 4, three hundred and thirty-nine UEEs have been observed (85 in the DAX and 254 in the MDAX) during the 125 trading days of our sample (2.7 events on average per day). The number of events is significantly higher in the MDAX. This is consistent with the differences in liquidity and trading interest exhibited in Table 2. The difference in trading intensity between the DAX and the MDAX is also reflected in the higher average repetitions (7.918 vs 6.519).

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Table 4 Ultrafast extreme events (UEEs) summary

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Figure 2 shows the number of UEE occurrences per day. Extreme events happened in 102 out of 125 trading days (84.30%). UEEs have occurred on the DAX (MDAX) during 40 (94) days. The higher number of daily UEE occurrences in the MDAX reflects its thinner trading and its shallower depth of LOB level one compared to the DAX. Spikes in the number of UEEs do not happen simultaneously in both indexes. This suggests that their causes are idiosyncratic rather than systematic.

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Figure 2 Number of UEEs per day: DAX - MDAX

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Figure 3 displays the number of UEE occurrences per minute, considering the 510 minutes of trading on regular days. UEEs exhibit a tendency to occur around the open and the close of the day as the documented smile in trading volume (Hanif and Smith (2012), Madhavan (2002)). UEEs can result from induced uncertainty by the market model that imposes long lasting suspension of trading.

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Figure 3 Number of UEEs per minute: DAX - MDAX

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The assumption of independent arrivals of HFMM signals and LFTs market orders is implicit in the Poisson distributions used by Ait-Sahalia and Saglam (2014). The HFMM decision follows the arrival of new information (LFTs marketable orders or signals), so we test the independence assumption on the aggregated information. Table 5 shows that one cannot reject this assumption for any stock in our sample.

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Table 5 Signal and trade independence: chi-square tests

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5. TVWAR: a time- and volume-weighted average return

Data emulation replicates the stock behavior. The model adapts the OQP dynamically to the stock's states by tracking the parameters λ (the arrival rate of LFTs' market orders), μ (the arrival rate of HFMM's signals), and C (the bid-ask spread). This induces dynamic management of positions. To evaluate the HFMM's performance, we propose a measure based on realized PnL, which considers the impacts of leverage and asynchronous data. Both factors affect the holding period and the discrete time returns.

Equations **(3)** to **(6)** define the variables required to determine the holding period return of a sequence of E events. A profit (loss) is realized when an existing position (long or short) is unwound. The unwinding quantity comes from two sources: the HFMM's marketable orders due to risk management features and incoming market orders executed against the HFMM's LOs. The unwinding quantity refers to a traded quantity that partially or totally offsets a position.

The unwinding quantity is:

$$UQ_e = \begin{array}{cc} if\ Pos_{e-1} > 0\ and\ \ Pos_{e-1} > Pos_e & -min(Pos_{e-1} - Pos_e, Pos_{e-1}) \\ if\ Pos_{e-1} < 0\ and\ \ Pos_{e-1} < P1os_e & -max(Pos_{e-1} - Pos_e, Pos_{e-1}), \\ else & 0 \end{array} \quad \textbf{(3)}$$

where:

UQ_e: unwinding quantity for event e, negative (positive) for buys (sells).
Pos_e: quantity long, short or flat for event e.
e: event number (an event = a trade).
$e \in [1,2, \dots, E]$.

Maximum leverage ensues from two factors: the OQP, which depends on the parameters (Γ, D, λ, μ, C, P) of Ait-Sahalia and Saglam (2014) and κ, the reference quantity defined in Section 3. Effective leverage, with an upper bound equal to the maximum leverage, is influenced by speed (latency, time priority, and market-making competition), HFMM LOs, and incoming market orders (quantity and serial correlation).

The effective leverage value is:

$$\Phi_e = \left(\frac{UQ_e}{\kappa/2}\right), \quad \textbf{(4)}$$

where Φ_e is based on the required capital to trade κ shares considering standard margin requirements. An unwinding trade for the HFMM resulting in a partial execution of the reference quantity κ has a leverage value smaller than the leverage value of an unwinding trade that closes the HFMM position of 2κ. 2κ is possible when the OQP threshold is 2 and the HFMM carries the maximum inventory.

The holding period return of event e is:

$$r_e = ln\left(\frac{P_e}{P_{e-1}}\right) \cdot \Phi, \qquad \textbf{(5)}$$

where P_e is the trade price for event e.

Returns are directly impacted by the relative importance of the unwinding trades. All else being equal, there is a linear relationship between the leverage measure and the holding period return of an event.

The cumulated return over E events is:

$$r_E = \sum_{e=1}^{E} r_e. \qquad \textbf{(6)}$$

Asynchronous events are the norm in microsecond trading environments. Significant differences exist in stocks' behavior due to their liquidity and depth, and to the trading interest. To facilitate comparison, we use discrete time intervals where the returns are time-weighted and volume-weighted within the interval. Equations **(7)** to **(14)** define the variables required to determine the discrete time return over D time intervals.

If a position overlaps two or more time intervals, the return is evenly spread out over the holding period. The number of time intervals in a trading day is equal to:

$$D = \lceil(\mu S_{eod} - \mu S_{bod})/\mu SbyInterval\rceil, \qquad \textbf{(7)}$$

where:

$d \in [1,2, \dots, D]$.
μS_{eod}: time stamp of the end of the day in µS.
μS_{bod}: time stamp of the beginning of the day in µS.
$\mu SbyInterval$: number of µS in a one-time interval.

The reference time stamp at the beginning of period d is given by:

$$\mu SBegRef_d = \mu S_{bod} + (d-1) \cdot \mu SbyInterval. \qquad \textbf{(8)}$$

The reference time stamp at the end of period d equals:

$$\mu SEndRef_d = \min(\mu S_{bod} + d \cdot \mu SbyInterval, \mu S_{eod}). \quad \textbf{(9)}$$

The reference time stamp at the beginning of period d for event *e* solves:

$$\mu SBeg_{d,e} = max(\mu S_{d,e}, \mu SBegRef_d), \quad \textbf{(10)}$$

where:

$\mu S_{d,e}$: time stamp of event e in time interval d.

The reference time stamp of period *d* for event *e* is bounded by the end of the time interval *d*, so the reference time stamp at the end of period d for event *e* is:

$$\mu SEnd_{d,e} = min(\mu S_{d,e}, \mu SEndRef_d). \quad \textbf{(11)}$$

Our procedure allows us to consider both the leverage $\left(\frac{UQ_{d,e}}{\kappa/2}\right)$ and the holding period $\left(\frac{\mu \mathrm{SEnd}_{d,e} - \mu \mathrm{SBeg}_{d,e}}{\mu SByInterval}\right)$ of positions. The time- and volume-weighted return of event e during time interval d is:

$$r_{d,e} = \ln\left(\frac{P_{d,e}}{P_{d,e-1}}\right) \cdot \frac{UQ_{d,e}}{\kappa/2} \cdot \left(\frac{\mu \mathrm{SEnd}_{d,e} - \mu \mathrm{SBeg}_{d,e}}{\mu SByInterval}\right). \quad \textbf{(12)}$$

The return for time interval d is the sum of the time- and volume-weighted returns of the *E* events of period *d*:

$$r_d = \sum_{e=1}^{E} r_{d,e}. \quad \textbf{(13)}$$

The cumulated return over *D* time intervals adjusted for leverage and holding periods equals:

$$r_D = \sum_{d=1}^{D} r_d. \quad \textbf{(14)}$$

6. Results

We emulate the trading and quoting activities of an HFMM under two scenarios: Section 6.1 implements the OQP designed by Ait-Sahalia and Saglam (2014) dynamic inventory model. In contrast, we close all positions by issuing market orders before the end of the day. In that way, the results are as representative as possible of "pure" market-making activities. Whereas Ait-Sahalia and Saglam (2014) show that an HFMM who holds private information can exploit it to his advantage, we do not consider this opportunity because we do not want to interfere in the price discovery process. A latency of 150 microseconds is applied to take into account the cycle of reception, analysis and response from our infrastructure. The worst-case scenario is applied to the HFMM time priority as defined in Section 3. In Section 6.2, we relax the constraint on the quantity of each order fixed at 1 by using κ. The HFMM does not optimize on the quantity in each trade because κ is constant. The management of positions considers the dynamic nature of the OQP, the trading intensity and the liquidity needs of the market participants. Circuit-breakers are implemented and are based on the monitoring of market characteristics summarized by the arrival rates of new information (trades and quotes) and the behavior of the bid-ask spread. They involve the use of market orders. We avoid price impacts by restricting the quantity of market orders to the available quantity at level one. This forces the HFMM to incur market risk instead of transferring the liquidity risk to the other market participants by walking into the LOB. We analyze the impact of circuit-breakers on profitability and market quality. Profitability measures are from the Profit and Loss (PnL) report, which is calculated from all HFMM orders (limit and market). The approach to assess the PnL meets the requirements of the Basel Committee on Banking Supervision (2013).

6.1 Ait-Sahalia and Saglam (2014)

Figure 4 displays the aggregated cumulative Profit and loss from the quoting and trading activities based on Ait-Sahalia and Saglam (2014) dynamic inventory model. Both indexes exhibit an upward trend over the entire period without significant drawdowns. The OQP allows the HFMM to extract a constant annuity from both indexes.

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Figure 4 DAX - MDAX Cumulative P&L: OQP
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Daily and intraday statistics are presented in Table 6. Total profits for the six-month period are 3,412 € (2,999 €) in the DAX (MDAX). Total profits are 14.8% higher in the DAX than in the MDAX. This is the result of a higher number of trades, 181,594 vs 86,199 for the DAX vs. the MDAX combined with a tiny average profit per trade of 0.019 € vs 0.034 €. This is characteristic of high-frequency trading (Jones (2013)). Total profits are insufficient to maintain the infrastructure costs required by market-making activities.

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Table 6 Daily and intraday profitability – OQP
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6.2 Trading strategy

The dynamic management of the OQP thresholds coupled with the use of κ based on the idiosyncratic characteristics of the stocks expose the HFMM's LOs to partial executions. Table 7 illustrates the way partial execution of the HFMM's limit orders against marketable orders are handled. The example comes from the emulation of Deutsche Bank data from February 1, 2013. It illustrates the way partial executions of the HFMM's limit orders against marketable orders are handled. It works as follow: One µS before ….054552, the HFMM is short 500 shares. At ….054552, a bid limit order creation for 1,000 shares at 42.910 is sent to the Exchange. We identify this order with the internal id 41. Internal ids refer to emulated orders from the trading strategy. At ….139361, an incoming market order hits the HFMM bid for a quantity of 169 (internal id 7).

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Table 7 Orders and positions: an example
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The HFMM position is short 331 shares. This trade is immediately followed by the cancellation of the LO with id 41, a creation (id 42) of a bid LO at 42.910 for 831 shares and a creation of an ask LO at 42.950 for 169 shares. Considering the HFMM's position (short 331) and his LO quantity of 831 (169) on the bid (ask), a full execution of one of his LOs will lead to the HFMM's κ of 500 shares for Deutsche Bank. This quantity is the maximum of the five most traded quantities of Deutsche Bank as defined in Section 3.

The profitability of the strategy is displayed in Table 8. Panel A indicates that total profits are strongly positive. The average daily profits are economically significant, the standard deviations low, and no daily loss has occurred over the 125-day period even if the sample includes different kinds of market moods, including some high stress periods (See Figure 1 DAX daily quotes - February to July 2013).

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Table 8 Daily and intraday profitability
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Panel B shows an average profit per trade of 2.32€ (1.73€) for the DAX (MDAX). When coupled with the total number of trades of Panel C, we obtain results typical of high-frequency trading: a high number of trades (1.1m for the DAX and 355.5k for the MDAX) paired with a small profit per trade. This highlights the fact that the HFMM is not a directional trader. Using hard information, he benefits from the bid-ask bounces and the variability of bid-ask spreads due to varying liquidity levels without assuming the use of valuable private information. The lowest part of Panel C displays the distribution of trade profit per transaction. Whereas flat trades represent 20.53% (15.75%) of the 1.1 million (350k) trades, both distributions are skewed to the right. An HFMM acting as a designated sponsor and using the strategy has his transaction fees waived because he fulfills the Deutsche Boerse (2015) requirements.

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Table 9 HFMM's Trade origins
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Looking at Table 9, a total of 1.1m (357k) trades have been executed in the DAX (MDAX) segment. 94.54% (89.13%) of these originate from HFMM LOs, which have been executed against incoming marketable orders from LFTs and other high-frequency traders, thus providing 15.3b € (1.6b €) in liquidity over the period. This confirms the capacity of the trading algorithm to act as a market maker.

Circuit-breakers (C.B.) are important components of the strategy. They have been activated a daily average of 14.5 (7.7) times per stock ($DAX: 54{,}462 \div 125 \div 30$). Looking at the DAX components, market orders triggered by the circuit-breakers represent 4.87% of total trades and they account for 9.94% of the total PnL. Results are even more striking when looking at the MDAX: 8.07% of the trades come from the circuit-breakers for a contribution exceeding 30% of the total PnL. Avoiding overnight positions comes at a cost: respectively 135,334 € and 15,086 € for the DAX and MDAX components.

Traditionally, informed traders are associated with high-volume transactions: Blume, Easley, and O'Hara (1994) show that volume provides information on information quality. Wang (1994) finds that volume is positively correlated with absolute changes in prices and dividends. Chakravarty, Gulen, and Mayhew (2004) relate informed trading in both stock and option markets to trading volume. However, developments in market structure and technological advances led to evidence of dynamic use of limit order strategies by which traders manage their positions: Bloomfield, O'Hara, and Saar (2005) use experimental asset markets to analyze make-take decisions in an electronic market. They note that informed traders' aggressive orders are replaced by limit orders as prices move toward fundamental values. Hasbrouck and Saar (2009) find evidence consistent with the use of a dynamic limit order strategy by which traders manage their positions on INET.

To investigate the impact of informed traders on HFMM's performance in this context, we compare the returns using a constant leverage to the discrete time returns adjusted for leverage and asynchronous trades using Equations **(7)** to **(14)**. If large trades convey private information, the HFMM's performance adjusted for leverage should decline to reflect the permanent impact on fundamental value of the private information. The results in Table 10 mitigate this conclusion. In the DAX, the cumulated leveraged return is 99.9% higher than the cumulated constant return. This can be the result of mixed strategies using market and limit orders described by Easley, de Prado, and O'Hara (2016). This casts doubts on the quality of information obtained from trade volume alone. *A contrario*, the difference between the leveraged and constant return is –19.5% in the MDAX. The trade-off between market risk and liquidity risk in this segment could explain this phenomenon.

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Table 10 Impact of leverage on performance
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Figure 5 displays the discrete time returns per time interval adjusted for leverage and asynchronous trades using Equations **(7)** to **(14)**. Dynamic management of positions is well suited to benefit from UEEs because the best discrete returns are obtained at the opening and the closing of the trading session, where the majority of UEEs happen. Moreover, time intervals 330-331 (327-329) and 420-423 (417-418) in the DAX (MDAX) coincide with a larger than usual number of UEE occurrences. The Deutsche Boerse market model, implying midday auctions, imposes a transfer of wealth between LFTs toward the HFMM as seen during period 243 (13h03) in the DAX: it exhibits a significant increase in average leveraged return for the HFMM. The same phenomenon is observed in the minute following the midday auction in the MDAX (period 246 at 13h06).

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Figure 5 DAX- MDAX Leveraged return per time interval
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The HFMM is a liquidity provider when he behaves as a market maker and issues market orders (consumes liquidity) for risk management purposes. Table 9 divides the HFMM's trade origins into three categories: the execution of the HFMM's LOs against incoming LFTs' marketable orders (LOB); the intraday liquidation of the HFMM's positions due to circuit-breakers (C.B.); and the closing of positions at the end of the day (O/N). This disentangles the liquidity-providing activities (LOB) from the liquidity-consuming ones (C.B. and O/N).

More than 94% (89%) of trades in the DAX (MDAX) originate from LOs. This confirms the HFMM's role as a liquidity provider. The monitoring of market condition has triggered 54.4k (28.9k) market orders in the DAX (MDAX). Nevertheless, the average profit per trade in both indexes is more than twice that obtained by LOB activities. An explanation is linked to the bid-ask spread: as volatility increases with extreme market conditions, the bid-ask spread widens. This has a direct and positive impact on the profitability of liquidated positions. Closing positions at the end of the day required an average of 1.75 (2.0) trades in the DAX (MDAX).

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Table 11 Participation in trades

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As depicted in Table 11, the HFMM would have had a positive impact on the market. He has traded €17.7b of market value, more than 90% of which comes from limit orders.

7. Robustness tests

7.1 Speed's impact on performance

Latency, the required time to receive, process, and react to new information, is considered crucial to the HFMM. Short latencies allow to limit being sniped on stale quotes, to aggregate new information rapidly and to gain access to market orders via time priority. Testing the effect of latency on performance is equivalent to quantify the impact of investments in technological infrastructures and softwares. It can serve as benchmark either to compare traders which differ solely by their speed or for capital budgeting decisions.

The treatment time of new information by the HFMM's strategy is 104 µS. The lower bound of the latencies analyzed is 150 µS to allow for order transmissions to the Exchange. We consider that the reference HFMM is using colocation facilities. To test for the impact of latency on the stragegy, we use latencies of 150, 500, 1 500, 5 000, and 10 000 µS. For the sixty stocks over the six-month period, we emulate real-time trading to obtain all limit and market orders. We calculate intraday and daily PnL. Finally, we aggregate the statistics by index. Results are presented in Table 12.

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Table 12 Impact of Latency on Performance
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For the DAX, decreasing latency does not influence the relative risk: $\sigma(\pi)/\pi$ is constant throughout all level of latencies. The investment in colocation services is fully justified by the augmentation of € 342 000 in total profits generated by the strategy between 10 000 µS and 150 µS latencies[3].

7.2 Strategy's features

Financial engineering is needed when designing a trading algorithm. Table 13 presents the results obtained by emulating the HFMM's market-making activities for the thirty DAX`s components during February to July 2013. The results of each column differ by the algorithm's features. The OQP from Ait-Sahalia and Saglam (2014) is the kernel of the HFMM's quoting decisions. It allows the obtaining of positive cash flows throughout the analyzed samples (ref. Section 6.1). However, their model of dynamic inventory management uses a quantity of one. This imposes a constraint on the profitability which is insufficient to cover the infrastructure costs required by the market-making activities. Setup 1 relaxes this constraint using the kappa concept defined in Section 3. It does not consider circuit-breakers or closing positions at the end of the day. The profitability of the period reached almost € 2.7 million and the average daily profit was € 21,569. However, the largest daily loss is € 853,324 and the daily volatility is excessively high (€ 235,856). Moreover, the distribution of profit and loss by transactions is highly leptokurtic. These characteristics are inconsistent with those expected from a market-maker (Brogaard et al. (2016)). Setup 2 adds the circuit-breakers.

---------- insert ----------
Table 13 Impact from strategy's features
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It results in a substantial increase in profitability (to € 3.2 million). The maximum daily loss, volatility and distribution of PnL per transaction still do not match the behavior of a market-maker. Setup 3 requires the closing of positions at the end of the day without the inclusion of circuit breakers. Total profits are € 1.4 million and significant changes have occurred in risk features. The maximum daily loss drops to € 3,488 and volatility to € 5,260. Setup 4 which includes kappa, circuit-breakers and EOD liquidations is the proposed trading strategy from Section 3. Profitability

[3] 10 Gbits/s connections are available in data center in Frankfurt/Main, Germany for a monthly fee of € 4 500 ref: Deutsche Boerse (2015).

exceeds € 2.7 million, no daily loss is incurred, and the distribution of the PnL fits a market maker's behavior. Results are qualitatively the same when analyzing the thirty components of the MDAX.

8. Conclusion

We have implemented the optimal quoting policy of liquidity provision of Ait-Sahalia and Saglam (2014) without the assumption of valuable private information. Profits in both DAX and MDAX over the six-month period exhibit an upward trend without significant drawdowns. The OQP allows the HFMM to extract a constant annuity from the market. However, total profits are insufficient to maintain the infrastructure required by market-making activities.

We have embedded the OQP in a trading strategy. The trading strategy avoids data mining and data snooping. It considers latency and partial executions of limit and market orders. Special care has been taken to eliminate the price impact linked to the HFMM's trading and quoting activities. Circuit-breakers have been implemented in response to regulators' concerns.

The viability of the strategy has been established using an extensive dataset including sixty stocks in two market segments. It covers a six-month period where the market has encompassed drastically different phases. The strategy exhibits outstanding characteristics when risk and profitability are considered. Market-making activities in both indexes led to 3.45 million € in profit for the six-month period. This is realized through 1.5 million trades, and no daily loss is incurred for either of the indexes. These results are the lower bound of the potential HFMM's performance considering: 1) the worst case scenario applied to his time priority, 2) the way partial executions are handled (loss of time priority, delay to cancel and re-enter quotes), 3) the constraint to incur market risk when using market orders implying quantities exceeding the available ones at level 1, 4) no informational advantage, 5) no maker-fee revenues, and 6) no valuable private information. We have disentangled the liquidity-providing role from liquidity-consuming activities. Whereas the core of the profits comes from quoting activities, the implementation of circuit breakers adds to total profits. Avoiding overnight positions focuses the appraisal on market-making instead of on directional trading.

Ultrafast extreme events have been documented; they occur regularly. They are linked to idiosyncratic characteristics and do not exhibit systematic behavior. In a context where price impact minimization is a major concern for traders, the prevalence of UEEs deserves further research. These high-frequency events could be associated with elusive liquidity, predatory behaviors, algorithm glitches, and aggregation of information.

Because partial executions of orders (limit and market) are possible, we have proposed a methodology to determine the returns that simultaneously take into consideration the varying leverage and the asynchronous nature of high-frequency trading. This procedure lets one quantify the impact of high volume trades often attributed to informed traders. The effect of high volume trades on the HFMM's performance is inconclusive. Aggregated results show that the HFMM's performance increases in the DAX and decreases in the MDAX. The HFMM behaves like a constant liquidity provider and has a positive effect on market quality.

As expected, latency affects performance. Investments in infrastructure and softwares are warranted by the increase in profitability and the HFMM can exploit his speed's advantage to economically significant levels. Empirical research must adress market imperfections as they have considerable impact on both risk and profitability. Implementing circuit-breakers and closing end of day positions are crucial to the economic viability of the HFMM.

Ait-Sahalia and Sağlam (2016) have recently published an extension to the model analyzed in this paper. They endogenize the bid-ask spread which is a function of the HFMM's quoting decisions. These decisions are driven by a signal about the likely type of trader (patient or impatient) who will send the next incoming marketable order. To test adequately this new setup, one has to quote and trade actively in live markets as it involves both liquidity provision and supply curve decisions which are modifying the state of the LOB and the price discovery process. No academic financial laboratory is available around the world (Lopez De Prado (2015)) to realize that kind of test.

9. Appendix Threshold calculation: An efficient algorithm

Output: L^*, U^*

Initialize $L = 1$ and $flag = 0$

While $flag = 0$ do

$U \leftarrow 1$;

While $U \leq -L$ do

Solve for $v(L-1,1), v(L,1), \cdots, v(-L,1), v(-L+1,1)$;

if

$$v(L,1) - v(L-1,1) > \frac{c}{2\delta}, v(L+1,1) - v(L,1) \leq \frac{c}{2\delta}, v(U,1) - v(U+1,1) > \frac{c}{2\delta}, v(U-1,1) - v(U,1) \leq \frac{c}{2\delta}$$

Then

$flag \leftarrow 1, L^* \leftarrow L \; and \; U^* \leftarrow U$;

Break;

$U \leftarrow U + 1$;

$L \leftarrow L - 1$;

Where:

L: Lower bound threshold.
U: Upper bound threshold.
$v(L|U, s)$: Value function at threshold L or U, having a signal s
$s \in \{1, -1\}$. 1 denotes a buy signal and -1 denotes a sell signal.
$c = \delta C$.
C: Bid-ask spread.
$\delta = \frac{\lambda+\mu}{\lambda+\mu+D}$.
λ: Arrival rate of LFTs market orders
μ: Arrival rate of HFMM signals s.
D: Constant discount factor > 0.
Γ: Inventory aversion coefficient.

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Table 1 Market summary statistics, trades and LOB1

Panel A					
Trades	**Market Value (MV)**	**% MV**	**# Trades**	**% Trades**	
DAX (30)	€ 398,504,790,578	92.57%	17,637,381	79.47%	
MDAX (30)	€ 31,986,673,636	7.43%	4,557,183	20.53%	
Total	**€ 430,491,464,214**	**100.00%**	**22,194,564**	**100.00%**	
Panel B					**# UTD/**
LOB, level 1	**Market Value (MV)**	**% MV**	**# UTD**	**% UTD**	**# Trades**
DAX (30)	€ 11,038,241,222,840	95.48%	207,225,811	80.96%	11.75
MDAX (30)	€ 522,821,561,308	4.52%	48,740,327	19.04%	10.70
	€ 11,561,062,784,148	**100.00%**	**255,966,138**	**100.00%**	**11.53**

The data span is from February 2 to July 30,2013. #UTD/#Trades are used to monitor high frequency trading activities, as for MIDAS, Security and Exchange Commission (SEC) at http://www.sec.gov/marketstructure/midas.html.

Table 2 Two-way classification of price movements in consecutive intraday trades: Summary (,000)

		+,+	+,0	+,-	0,+	0,0	0,-	-,+	-,0	-,-	Total
DAX	**Occ.**	1,253.8	1,776.1	1,599.7	1,781.8	4,824.6	1,770.0	1,594.0	1,775.7	1,261.7	17,637.4
	%	7.1%	10.1%	9.1%	10.1%	27.4%	10.0%	9.0%	10.1%	7.2%	100.0%
MDAX	**Occ.**	418.0	445.0	478.6	443.9	988.7	442.3	479.6	441.2	420.0	4,557.2
	%	9.2%	9.8%	10.5%	9.7%	21.7%	9.7%	10.5%	9.7%	9.2%	100.0%

Price movements are classified into “up” (+), “unchanged” (0), and “down” (-). Price moves are represented by x,y where x is the i^{th-1} move and y the i^{th} move. % is the relative occurrence of the column’s price movement.

Table 3 Two-way classification of price movements in consecutive intraday trades DAX (%)

Isix	+,+	+,0	+,-	0,+	0,0	0,-	-,+	-,0	-,-
22	8.36%	10.12%	9.63%	10.22%	23.33%	10.12%	9.53%	10.22%	8.47%
24	8.90%	10.64%	8.81%	10.63%	22.20%	10.58%	8.82%	10.57%	8.86%
32	9.51%	9.78%	9.47%	9.70%	23.74%	9.56%	9.55%	9.48%	9.23%
49	7.22%	10.59%	8.80%	10.59%	24.94%	10.84%	8.79%	10.84%	7.38%
58	8.61%	10.44%	9.40%	10.52%	22.47%	10.32%	9.32%	10.40%	8.53%
60	9.72%	10.39%	10.17%	10.32%	19.36%	10.13%	10.24%	10.05%	9.63%
80	3.23%	10.13%	6.04%	10.36%	40.89%	10.01%	5.80%	10.25%	3.28%
85	3.92%	10.41%	5.97%	10.33%	38.66%	10.36%	6.05%	10.28%	4.02%
106	5.94%	10.44%	8.16%	10.59%	29.99%	10.43%	8.02%	10.58%	5.85%
130	4.06%	10.10%	6.98%	10.38%	37.63%	9.88%	6.70%	10.16%	4.11%
138	2.14%	10.29%	7.66%	10.03%	39.89%	9.99%	7.93%	9.73%	2.34%
143	5.01%	10.64%	6.83%	10.44%	33.72%	10.74%	7.03%	10.54%	5.06%
146	1.97%	8.84%	5.42%	8.98%	49.76%	8.80%	5.28%	8.94%	2.02%
151	3.45%	10.01%	6.25%	10.28%	40.55%	9.90%	5.97%	10.18%	3.41%
266	7.55%	9.96%	10.34%	10.16%	24.34%	9.79%	10.14%	9.99%	7.74%
829	8.96%	10.00%	10.11%	9.97%	22.06%	9.98%	10.14%	9.94%	8.83%
1634	10.72%	9.95%	10.78%	10.01%	17.47%	9.63%	10.72%	9.69%	11.04%
2451	8.99%	9.65%	10.76%	9.51%	21.45%	9.79%	10.91%	9.64%	9.30%
2481	5.43%	10.45%	8.01%	10.49%	31.47%	10.37%	7.97%	10.41%	5.39%
2807	7.30%	10.42%	9.36%	10.22%	25.09%	10.49%	9.56%	10.29%	7.28%
2841	8.46%	9.92%	10.34%	9.89%	22.13%	10.14%	10.38%	10.10%	8.64%
3446	8.71%	10.29%	9.63%	10.24%	22.51%	10.12%	9.67%	10.08%	8.74%
3679	7.33%	9.79%	9.70%	9.75%	26.56%	9.92%	9.74%	9.87%	7.35%
3744	2.65%	9.60%	6.44%	9.78%	43.44%	9.46%	6.26%	9.64%	2.74%
4423	8.84%	10.13%	10.25%	10.30%	21.33%	10.03%	10.08%	10.20%	8.84%
5830	8.86%	10.19%	10.21%	10.16%	21.35%	10.15%	10.24%	10.12%	8.72%
8669	9.80%	9.92%	10.53%	9.87%	20.13%	9.83%	10.57%	9.78%	9.57%
9633	7.72%	10.12%	9.98%	10.33%	23.91%	10.10%	9.78%	10.30%	7.77%
11814	4.58%	10.34%	7.52%	10.40%	35.07%	10.00%	7.46%	10.07%	4.55%
16753	7.21%	10.24%	9.24%	10.34%	26.09%	10.12%	9.14%	10.22%	7.40%

Isix: unique stock identifier. Price movements are classified into "up" (+), "unchanged" (0), and "down" (-). Price moves are represented by x,y where x is the i^{th-1} move and y the i^{th} move. % is the relative occurrence of the column's price movement. Each row sums to 1. The data span is from February 2, 2013 to July 30, 2013.

Table 4 Ultrafast extreme events (UEEs) summary

	UEEs up				UEEs down				Total UEEs			
	# occ.	# stocks	avg. rep.	# days	# occ.	# stocks	avg. rep.	# days	# occ.	# stocks	avg. rep.	# days
DAX	33	18	8.333	23	52	23	7.654	32	85	26	7.918	40
MDAX	133	28	6.436	72	121	27	6.653	64	254	30	6.519	94
Total	**166**	**46**	**6.813**	**82**	**173**	**50**	**6.954**	**76**	**339**	**56**	**6.885**	**102**

UEEs up: surges in price; UEEs down: mini crashes in price; # occ: number of UEE occurrences; # stocks: number of stocks that experienced at least one UEE over the sample; avg. rep.: average number of successive tick up (tick down) by UEE; # days: number of days with at least one UEE.

Table 5 Signal and trade independence: chi-square tests

	DAX				MDAX		
Isix	X^2	**p_val**	**#/µS**	**isix**	X^2	**p_val**	**#/µS**
22	203,937,984	0.496	999,696	**39**	15,329,466	0.499	851,637
24	160,914,996	0.496	981,189	**54**	13,609,928	0.499	800,584
32	173,733,875	0.496	992,765	**63**	14,478,948	0.499	804,386
49	124,653,438	0.497	989,313	**68**	10,562,100	0.499	704,140
58	139,181,664	0.497	987,104	**86**	15,369,156	0.499	853,842
60	134,565,880	0.497	989,455	**95**	8,703,645	0.499	580,243
80	84,408,825	0.497	993,045	**98**	15,138,738	0.499	841,041
85	82,746,488	0.497	940,301	**112**	9,590,670	0.499	639,378
106	104,942,040	0.497	999,448	**117**	19,692,002	0.499	895,091
130	101,905,854	0.497	999,077	**177**	6,357,416	0.498	489,032
138	57,920,763	0.497	864,489	**661**	17,147,576	0.499	902,504
143	111,044,309	0.497	982,693	**1131**	10,887,405	0.499	725,827
146	144,863,120	0.497	999,056	**1415**	13,654,791	0.499	803,223
151	92,681,940	0.497	996,580	**1429**	14,507,892	0.499	805,994
266	89,374,050	0.497	993,045	**1457**	15,034,662	0.499	835,259
829	125,122,410	0.497	993,035	**1468**	7,207,956	0.498	514,854
1634	125,912,901	0.497	976,069	**1566**	16,050,003	0.499	844,737
2451	156,993,249	0.496	999,957	**2323**	15,455,538	0.499	858,641
2481	98,013,663	0.497	990,037	**3290**	5,190,090	0.498	346,006
2807	85,972,566	0.497	999,681	**3849**	11,741,355	0.499	782,757
2841	143,999,856	0.497	999,999	**4035**	13,745,027	0.499	808,531
3446	176,847,628	0.496	993,526	**5566**	10,716,976	0.499	669,811
3679	144,857,175	0.497	999,015	**8650**	11,551,908	0.499	679,524
3744	131,883,576	0.497	999,118	**10658**	10,201,716	0.499	728,694
4423	115,978,239	0.497	991,267	**10938**	13,980,222	0.499	822,366
5830	174,965,700	0.496	999,804	**11426**	9,910,173	0.499	762,321
8669	131,157,551	0.497	986,147	**11475**	13,561,136	0.499	847,571
9633	145,996,204	0.497	999,974	**11607**	7,284,465	0.498	485,631
11814	105,096,684	0.497	982,212	**11644**	7,226,336	0.499	555,872
16753	133,886,502	0.497	999,153	**13469**	16,029,576	0.499	890,532

Isix: unique stock identifier; X^2: chi-square test value; p_val: p_value; #/µS: number of microseconds with at least one signal.

Table 6 Daily and intraday profitability – OQP

Panel A	Daily Statistics			
		DAX (30)		MDAX (30)
Total	€	3,412	€	2,999
Avg	€	27.30	€	23.99
Min	€	(33.49)	€	(29.62)
Max	€	103.84	€	125.54
Std. dev.	€	23.77	€	16.39
Days		125		125
Panel B	Intraday Statistics			
		DAX (30)		MDAX (30)
# trades		181,594		86,199
Avg π/trade	€	0.019	€	0.034
Min	€	(13.35)	€	(14.00)
Max	€	2.00	€	10.90

Table encompasses results from February 2 to July 30 2013; Total: Total profit; Avg: Average profit; Min: Minimum profit; Max: Maximum profit; Std. dev.: standard deviation of daily profit; Days: Number of trading days; Avg π/trade: Average profit per trade; # trades: Total number of executed trades by the HFMM.

Table 7 Orders and positions: an example

Isix: 2481

Date	**Time**	**Type**	**Price**	**Q**	**Id**	**Pos**
.	.	.	.	.	.	-500
20130201	32901054552	1	42.910	1000	41	-500
20130201	32901139361	3	42.910	169	7	-331
20130201	32901139361	2	42.910	-1000	41	-331
20130201	32901139361	1	42.910	831	42	-331
20130201	32901139361	1	42.925	-169	-103	-331
20130201	32907274679	2	42.910	-831	42	-331
20130201	32907274679	1	42.890	831	43	-331
20130201	32907276733	2	42.925	169	-103	-331

Type: 1 = HFMM new limit order, 2 = HFMM limit order cancellation, 3 = incoming LFT market order executed against an HFMM limit order; Time stamps are in microseconds; q = order quantity; id: internal reference to algorithm activity; pos: HFMM position, negative values representing short positions.

Table 8 Daily and intraday profitability – Trading strategy

Panel A	**Daily statistics**			
	DAX (30)		**MDAX (30)**	
Total Profit	€ 2,765,462		€ 686,726	
Avg	€ 22,124		€ 5,494	
Min	€ 7,396		€ 1,035	
Max	€ 48,035		€ 13,805	
Std. Dev	€ 8,314		€ 2,349	
No. obs	125		€ 125	
	Intraday statistics			
Panel B				
Avg π/trade	€ 2.48		€ 1.94	
Panel C	**Distribution of trades per profit**			
	# trans	**% total**	**# trans**	**% total**
Total	1,113,352	100%	353,521	100.00%
<= -20	34,064	3.06%	17,227	4.87%
<-10 ; >= -20	51,135	4.59%	18,131	5.13%
<0 ; <= -10	268,015	24.07%	92,404	26.14%
0	228,603	20.53%	55,684	15.75%
>0 ; <= 10	333,731	29.98%	114,007	32.25%
> 10 ; <= 20	111,497	10.01%	28,545	8.07%
>20	86,307	7.75%	27,523	7.79%

Table encompasses results from February 2 to July 30 2013; Avg: Average daily profit; Min: Minimum daily profit; Max: Maximum daily profit; Std. dev.: standard deviation of daily profit; No. obs.: Number of trading days; Avg π/trade: Average profit per trade; Total: Total number of executed trades by the HFMM; <= -20, <-10 >-20, …, <=20: bins of number of trades with profit <= -20, <-10 >-20, …, <=20.

Table 9 HFMM's Trade origins

		# trades	% trades	MV (000 €)	% MV	P&L	% PL
DAX	**Total**	1,117,499	100.00%	€ 16,021,276	100.00%	€ 2,765,462	100.00%
	LOB	1,056,471	94.54%	€ 15,311,005	95.57%	€ 2,625,985	94.96%
	C.B.	54,462	4.87%	€ 642,880	4.01%	€ 274,811	9.94%
	O/N	6,566	0.59%	€ 67,362	0.42%	€ (135,334)	-4.89%
MDAX	**Total**	357,599	100.00%	€ 1,696,227	100.00%	€ 686,726	100.00%
	LOB	321,175	89.81%	€ 1,575,293	92.87%	€ 493,181	71.82%
	C.B.	28,860	8.07%	€ 97,511	5.75%	€ 208,631	30.38%
	O/N	7,564	2.12%	€ 23,399	1.38%	€ (15,086)	-2.20%

trades: HFMM number of trades; MV (000€): € market value of HFMM trades (in thousands); P&L: Profit (loss); LOB: HFMM limit orders executed against incoming LFTs' market orders; C.B.: circuit-breakers (HFMM market orders due to real-time monitoring of market conditions); O/N: HFMM market orders to flatten position overnight

Table 10 Impact of leverage on performance

DAX	Lev.	Constant		MDAX	Lev.	Constant	
Avg	**878.8%**	**778.9%**	**99.9%**	**Avg**	**557.2%**	**576.7%**	**-19.5%**
Isix	**Cumul**	**Cumul**	**Diff**	**isix**	**Cumul**	**Cumul**	**Diff**
22	968.4%	862.7%	105.7%	**39**	1034.4%	1003.9%	30.6%
24	408.1%	481.1%	-72.9%	**54**	172.6%	158.3%	14.3%
32	723.2%	855.8%	-132.6%	**63**	580.7%	542.8%	37.8%
49	585.3%	552.2%	33.1%	**68**	381.6%	357.8%	23.9%
58	690.5%	1053.0%	-362.5%	**86**	380.3%	488.2%	-107.9%
60	625.2%	742.6%	-117.4%	**95**	482.7%	231.6%	251.1%
80	751.3%	725.5%	25.8%	**98**	703.5%	813.0%	-109.6%
85	557.4%	413.5%	143.9%	**112**	549.4%	545.2%	4.1%
106	1327.3%	1191.0%	136.3%	**117**	719.9%	738.4%	-18.5%
130	1288.3%	1122.5%	165.8%	**177**	834.5%	522.9%	311.6%
138	756.1%	590.4%	165.7%	**661**	756.6%	698.8%	57.8%
143	712.4%	495.3%	217.1%	**1131**	686.9%	760.3%	-73.3%
146	1084.9%	786.9%	297.9%	**1415**	599.0%	644.5%	-45.5%
151	931.4%	731.3%	200.1%	**1429**	705.3%	653.2%	52.1%
266	327.0%	347.1%	-20.0%	**1457**	480.5%	487.4%	-6.9%
829	963.1%	636.5%	326.5%	**1468**	34.4%	47.3%	-12.9%
1634	520.9%	594.7%	-73.8%	**1566**	874.9%	754.1%	120.8%
2451	1357.5%	1462.5%	-105.0%	**2323**	479.2%	424.9%	54.3%
2481	96.7%	104.3%	-7.6%	**3290**	349.5%	582.0%	-232.4%
2807	963.2%	873.6%	89.6%	**3849**	853.2%	1210.0%	-356.8%
2841	1721.7%	1490.5%	231.2%	**4035**	239.5%	236.4%	3.1%
3446	829.4%	771.6%	57.8%	**5566**	700.3%	768.8%	-68.6%
3679	931.2%	716.5%	214.7%	**8650**	577.6%	616.9%	-39.2%
3744	1245.4%	834.7%	410.6%	**10658**	392.0%	288.6%	103.4%
4423	857.7%	818.4%	39.3%	**10938**	534.5%	434.5%	100.0%
5830	1171.5%	566.1%	605.4%	**11426**	704.9%	795.1%	-90.2%
8669	520.5%	617.9%	-97.4%	**11475**	967.6%	1492.3%	-524.6%
9633	1313.6%	1118.9%	194.7%	**11607**	298.0%	367.0%	-69.0%
11814	867.9%	753.3%	114.6%	**11644**	256.9%	260.1%	-3.2%
16753	1266.6%	1057.2%	209.4%	**13469**	385.2%	375.7%	9.5%

Lev. Cumul: cumulated return adjusted for leverage; Constant cumul: cumulated return with constant leverage.

Table 11 Participation in trades

		Realized	HFMM	HFMM/ Realized
DAX (30)	**Market Value**	€ 398,504,790,578	€ 16,021,291,071	**4.02%**
	# Trades	17,637,381	1,117,499	**6.34%**
MDAX (30)	**Market Value**	€ 31,986,673,636	€ 1,696,246,094	**5.30%**
	# Trades	4,557,183	357,599	**7.85%**

Realized column presents the summary statistics of realized market activities over the sample period (February 2 to July 30 2013). HFMM column presents what would have been the HFMM's activities over the same period.

Table 12 Impact of Latency on Performance

	Latency	10,000	5,000	2,500	1,000	500	150
DAX	$\boldsymbol{\pi}$	€ 2,578,400	€ 2,784,300	€ 2,860,600	€ 2,898,200	€ 2,913,199	€ 2,765,462
	$\boldsymbol{\bar{\pi}}$	€ 20,628	€ 22,274	€ 22,885	€ 23,816	€ 23,305	€ 22,124
	$\boldsymbol{min(\pi)}$	€ 8,182	€ 9,239	€ 8,799	€ 8,987	€ 8,952	€ 7,396
	$\boldsymbol{\sigma(\pi)}$	€ 7,609	€ 8,115	€ 8,420	€ 8,582	€ 8,635	€ 8,314
	$\boldsymbol{\sigma(\pi)/\bar{\pi}}$	0.37	0.36	0.37	0.36	0.37	0.37
MDAX	$\boldsymbol{\pi}$	€ 609,760	€ 662,230	€ 691,710	€ 709,940	€ 714,640	€ 686,726
	$\boldsymbol{\bar{\pi}}$	€ 4,878	€ 5,298	€ 5,534	€ 5,680	€ 5,717	€ 5,494
	$\boldsymbol{min(\pi)}$	€ 1,390	€ 1,724	€ 1,794	€ 1,783	€ 2,153	€ 1,035
	$\boldsymbol{\sigma(\pi)}$	€ 1,715	€ 1,787	€ 1,850	€ 1,893	€ 1,906	€ 2,349
	$\boldsymbol{\sigma(\pi)/\bar{\pi}}$	0.35	0.34	0.33	0.33	0.33	0.43

π: total profit; $\boldsymbol{\bar{\pi}}$: average daily profit; $\boldsymbol{\sigma(\pi)}$**:** standard deviation of daily profit; $\boldsymbol{min(\pi)}$**:** minimum daily profit

Table 13 Impact from strategy's features

DAX (30)		Setup 1	Setup 2	Setup 3	Setup 4
	tot profit	€ 2,696,098	€ 3,197,081	€ 1,435,176	€ 2,765,462
daily	**avg**	€ 21,569	€ 25,577	€ 11,481	€ 22,124
	min	€ (853,324)	€ (787,719)	€ (3,488)	€ 7,396
	max	€ 765,749	€ 553,144	€ 30,258	€ 48,035
	std dev	€ 236,586	€ 181,094	€ 5,360	€ 8,314
	# of days	125	125	125	125
trades	**avg**	€ 3.92	2.88	€ 2.07	€ 2.48
	min	€ (162,443)	€ (85,678)	€ (2,610)	€ (1,160)
	max	€ 58,471	€ 36,405	€ 1,866	€ 2,745
	std dev	€ 1,183.34	€ 703.62	€ 20.40	€ 17.17
	# trades	688,087	1,108,920	692,579	1,113,352
	<= -20	218,133	343,244	22,045	34,064
	<-10 ; >= -20	6,593	16,591	33,741	51,135
	<0 ; <= -10	29,582	59,467	177,390	268,015
	0	122,962	194,065	118,308	228,603
	>0 ; <= 10	30,986	61,595	217,977	333,731
	> 10 ; <= 20	8,399	20,469	71,408	111,497
	>20	271,432	413,489	51,710	86,307
	avg win	€ 402.35	€ 289	€ 10.44	€ 10.94
	avg loss	€ (481.15)	€ (334)	€ (9.12)	€ (8.64)
	avg w/l	0.84	0.87	1.15	1.27
	# win	310,817	495,553	341,095	531,535
	# loss	254,308	419,302	233,176	353,214
	# w/l	1.22	1.18	1.46	1.5

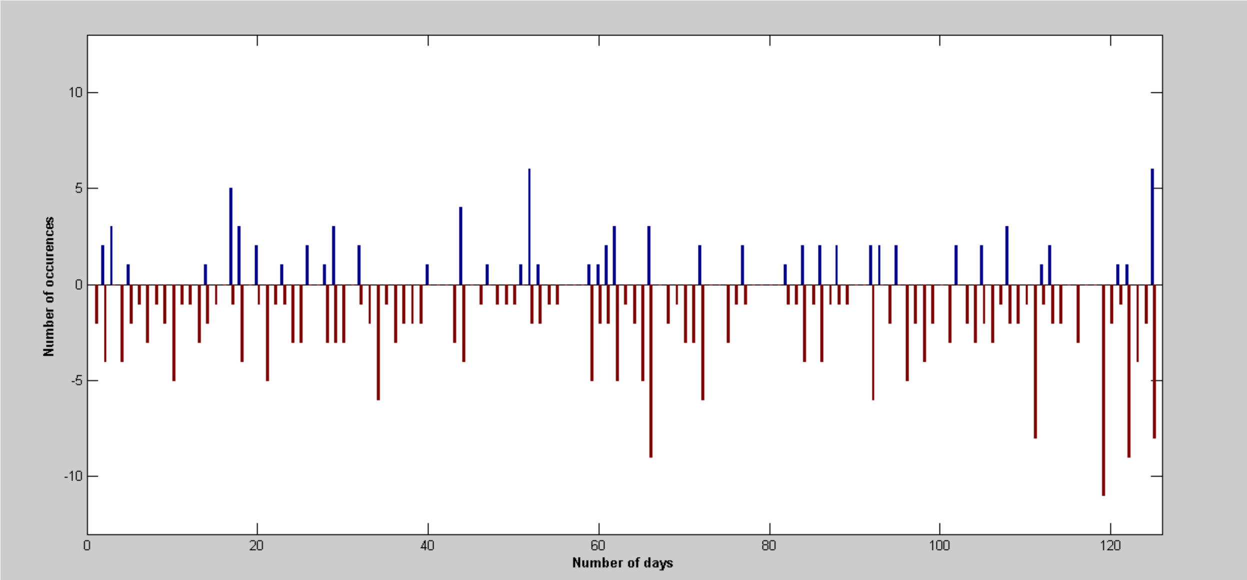
Setup 1: Circuit-breakers: off ; EOD liquidation: off ; Setup 2: Circuit-breakers: on ; EOD liquidation: off

Setup 3: Circuit-breakers: off ; EOD liquidation: on ; Setup 4: Circuit-breakers: on ; EOD liquidation: on

Figure 1 DAX daily quotes - February to July 2013

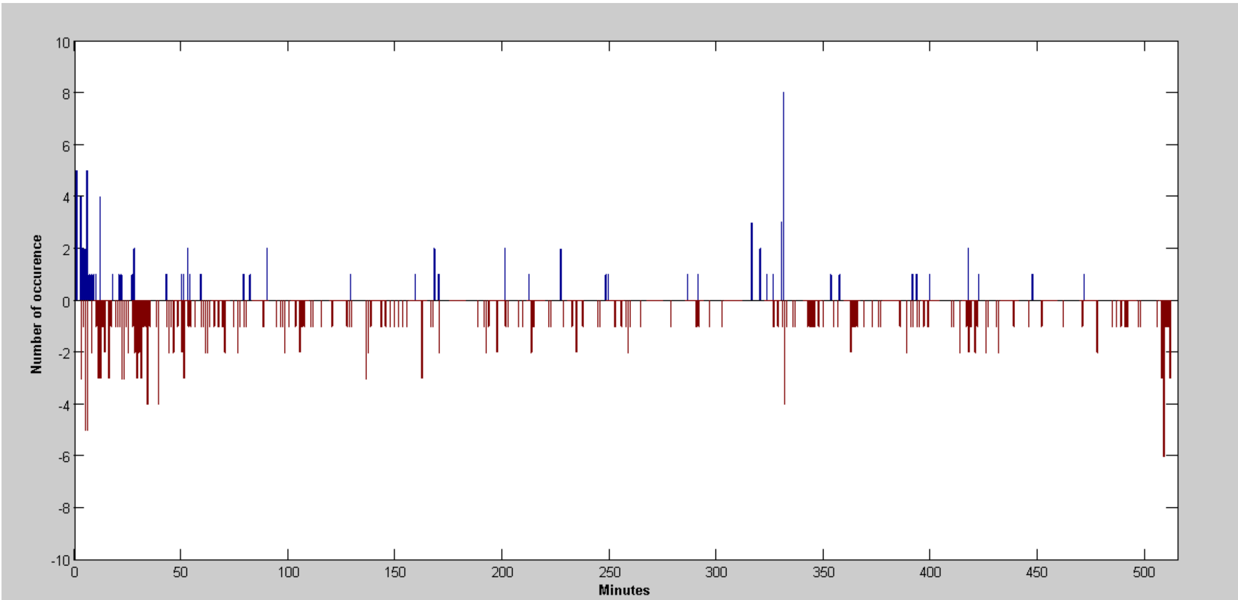
Trading range; Bear trend; Bull trend; High volatility

Figure 2 Number of UEEs per day: DAX - MDAX



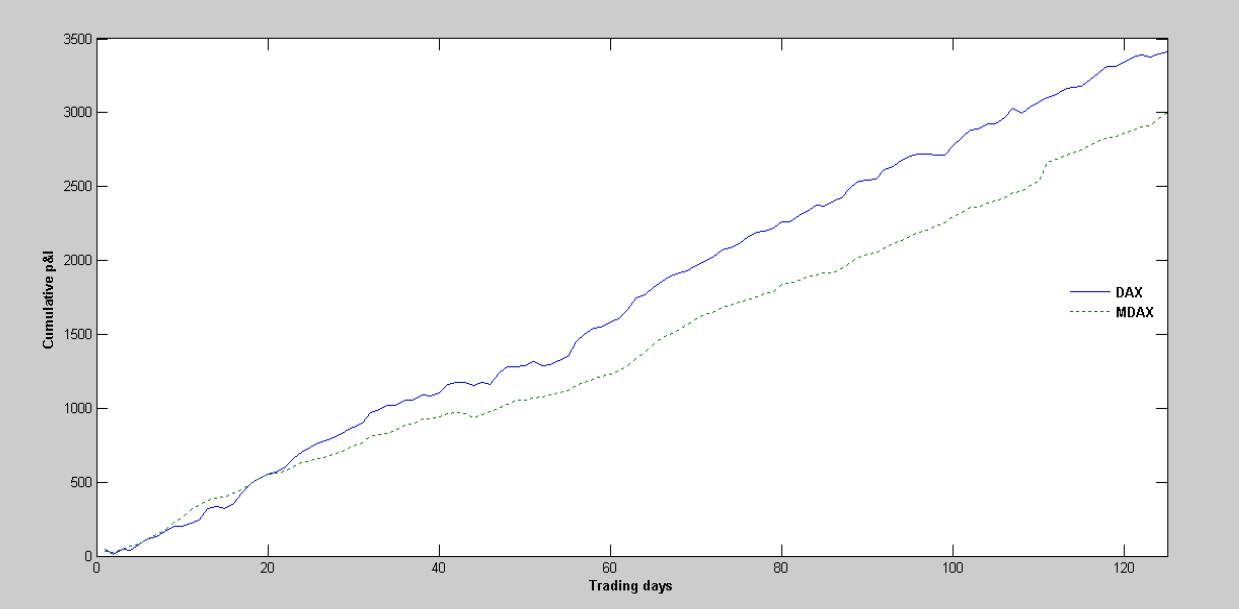
Horizontal axis: data sample of 125 trading days; vertical axis: number of occurrences of UEEs per day, DAX events are positive and MDAX events are negative for presentation purposes.

Figure 3 Number of UEEs per minute: DAX - MDAX



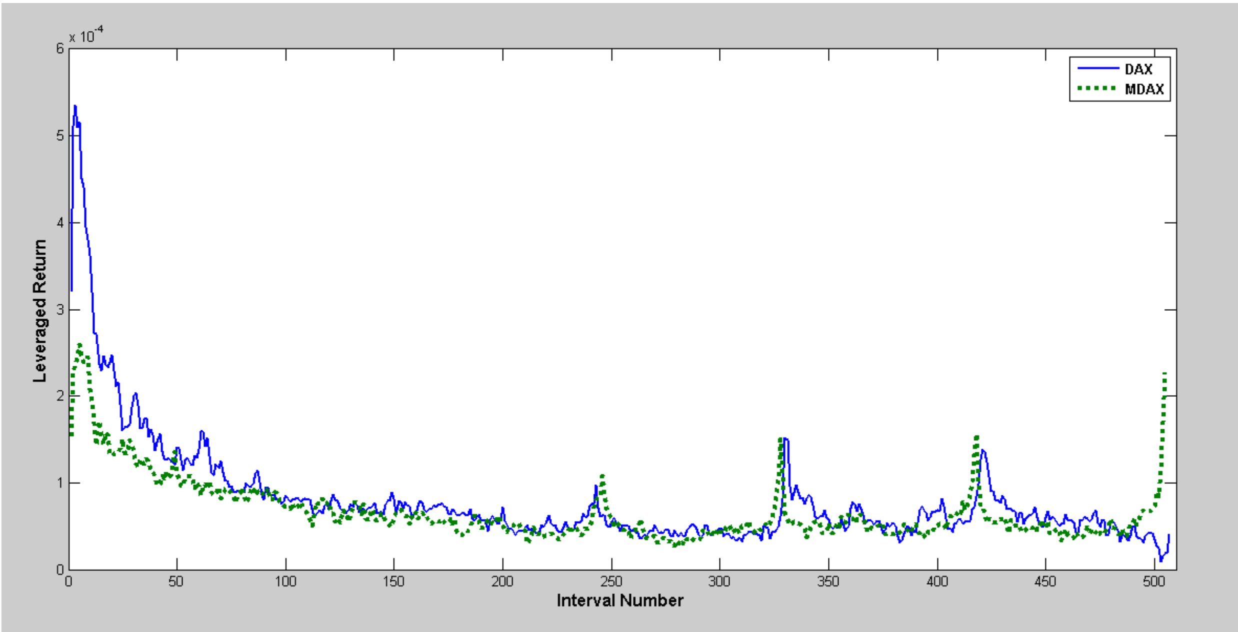
Horizontal axis: data sample of 510 minutes of trading per day; vertical axis: number of occurrences of UEEs per minute, DAX events are positive and MDAX events are negative for presentation purposes.

Figure 4 DAX - MDAX Cumulative P&L: OQP



Aggregated cumulative Profits & Losses obtained by Ait-Sahalia and Saglam (2014) Optimal Quoting Policy over the sample period (February 2 to July 30, 2013).

Figure 5 DAX- MDAX Leveraged return per time interval



One-minute time-volume weighted average returns (TVWAR) obtained by the trading strategy. Returns are aggregated by market indexes.

List of abbreviations

- **AT** algorithmic trading
- **DB** Deutsche Boerse AG
- **EOD** End of the day
- **HFMM** high frequency market maker
- **HFTers** high frequency traders
- **HFT** high frequency trading
- **LO(s)** limit order(s)
- **LOB** limit order book
- **LFTs** low frequency traders
- **MDP** Markov decision process
- **µS** microsecond
- **OQP** optimal quoting policy
- **PnL** Profit and Loss
- **TVWAR** time-volume weighted average return
- **UEE(s)** ultrafast extreme event(s)
- **UTD(s)** Update(s)